NONLINEAR CIRCUIT SIMULATION USING TIME-VARIANT FILTER

Jaromir Macak

Dept. of Telecommunications,
FEEC, Brno University of Technology
Brno, Czech Republic
jaromir.macak@phd.feec.vutbr.cz

Jiri Schimmel

Dept. of Telecommunications,
FEEC, Brno University of Technology
Brno, Czech Republic
schimmel@feec.vutbr.cz

ABSTRACT

The dynamic simulation of nonlinear guitar effects has recently been studied in depth. There are several approaches to the simulation of the distortion guitar effects. This paper presents an algorithm based on using a digital linear time-variant filter for the simulation of the nonlinear circuit of diode limiter. Filter coefficients are changed in each sample period according to the level of an input and output signal using numerical methods for the solution of nonlinear functions. The designed algorithm was used in the distortion effect to examine its characteristics. Sound examples of the implemented distortion effect can be found at web page www.utko.feec.vutbr.cz/~schimmel/DAFx09/.

1. INTRODUCTION

Real-time digital simulation of analogue guitar effects has recently become very popular. However, implementing these systems brings two contradictory requirements – accuracy versus computational complexity. Therefore the whole process of the simulation is divided into individual blocks and each block is simulated individually [1], [2]; this allows describing each block more precisely, but we have to ignore the effect of adjacent blocks. The analogue guitar distortion effects usually consist of filters and nonlinear blocks. The nonlinear block can be effectively implemented as a static waveshaper with good results [3]. Nevertheless, according to [4], a static nonlinearity doesn't work well on transients. A more accurate approach has been proposed in [3]. It is based on the solution of nonlinear ordinary differential equations (ODE) using implicit (Backward Euler) and explicit (Forward Euler, Runga-Kutta) solvers. Linear time-invariant (LTI) filters can be considered as solvers of linear ODEs and they can be used for the simulation of small-signal models of nonlinear systems because the coefficients of these models are constant. However, in the guitar distortion effect, large-signal models must be used. Large-signal models that do not have constant parameters, can be described by a set of small-signal models with different parameters. Therefore the linear time-variant filters must be used instead of the LTI filters.

2. DIODE LIMITER CIRCUIT MODEL

The diode limiter can be found in many guitar distortion effect pedals. Example of the diode limiter that provides one-way limiting is shown in Figure 1.The diode current is given by the equation

$$I_\mathrm{d} = I_\mathrm{s}(\mathrm{e}^{\frac{U_\mathrm{d}}{U_\mathrm{t}}} - 1), \tag{1}$$

where I_s is the saturation current, U_d is the voltage on the diode, and U_t is the thermal voltage. Using Kirchhoff's law we can obtain the nonlinear ODE

$$\frac{\mathrm{d}U_\mathrm{d}}{\mathrm{d}t} = \frac{U_\mathrm{i} - U_\mathrm{d}}{RC} - \frac{I_\mathrm{s}}{C}(\mathrm{e}^{\frac{U_\mathrm{d}}{U_\mathrm{t}}} - 1). \tag{2}$$

This equation can be solved using some of the methods for numerical solution of the ODE.

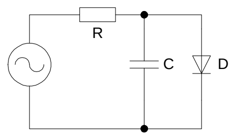


Figure 1: *Diode limiter model.*

2.1. Small-Signal Model of Diode Limiter

The diode from the circuit in Figure 1 can be replaced with a nonlinear resistance. This leads to the circuit in Figure 2. The nonlinear resistance r can be obtained from

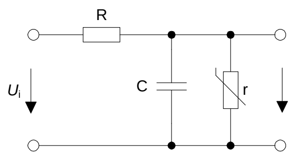


Figure 2: *Small-signal model of diode limiter.*

$$r = \frac{U_\mathrm{d}}{I_\mathrm{d}} = \frac{U_\mathrm{d}}{I_\mathrm{s}(\mathrm{e}^{\frac{U_\mathrm{d}}{U_\mathrm{t}}} - 1)} \tag{3}$$

and it is considered a constant in a small-signal model. Therefore it is possible to get the transfer function of this circuit

$$S(s) = \frac{r}{r + R + sCRr}. \tag{4}$$

The bilinear transform [5] of equation (4) results in

$$\begin{aligned} H(z) &= \frac{r + rz^{-1}}{r + R + 2f_\mathrm{s}CRr + (r + R - 2f_\mathrm{s}CRr)z^{-1}} \\ &= \frac{a_0 + a_1 z^{-1}}{b_0 + b_1 z^{-1}}, \end{aligned} \tag{5}$$

where a_0, a_1, b_0 and b_1 are the LTI filter coefficients. The output signal is then

$$U_\mathrm{d}[n] = \frac{a_0}{b_0}U_\mathrm{i}[n] + \frac{a_1}{b_0}U_\mathrm{i}[n-1] - \frac{b_1}{b_0}U_\mathrm{d}[n-1]. \tag{6}$$

2.2. Large-Signal Model of Diode Limiter with Linear Time-Variant Filter

A large-signal model works in a wider range of input voltages than the small-signal model, so the nonlinear resistance r is now a function of the output voltage U_d. The digital filter coefficients computed according to (5) are also functions of output voltage U_d. The output voltage equation (6) will change to

$$\begin{aligned} U_\mathrm{d}[n] = {} & \frac{a_0(U_\mathrm{d}[n-1])}{b_0(U_\mathrm{d}[n-1])}\,U_\mathrm{i}[n] + \\ & + \frac{a_1(U_\mathrm{d}[n-1])}{b_0(U_\mathrm{d}[n-1])}\,U_\mathrm{i}[n-1] - \\ & - \frac{b_1(U_\mathrm{d}[n-1])}{b_0(U_\mathrm{d}[n-1])}\,U_\mathrm{d}[n-1]. \end{aligned} \tag{7}$$

In this equation the output voltage value from the last iteration $U_\mathrm{d}[n-1]$ is used to compute filter coefficients $a_0(U_\mathrm{d}[n-1])$, $a_1(U_\mathrm{d}[n-1])$, $b_0(U_\mathrm{d}[n-1])$ and $b_1(U_\mathrm{d}[n-1])$. Then the new output signal value is computed. Figures 3 and 4 show the output signal for 1 kHz sinewave input with an amplitude of 1 V at a sampling frequency of 48 kHz. This algorithm needs to work at high sampling frequencies with view to the stability of the solution like explicit methods for solving the ODEs [3], [6].

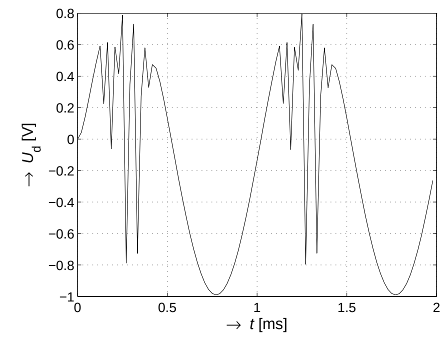


Figure 3: *Output voltage for 1 kHz sinewave input with amplitude of 1 V. No oversampling is used.*

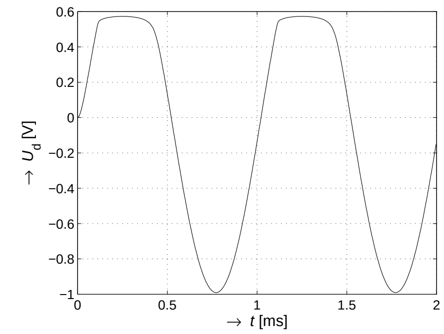


Figure 4: *Output voltage for 1 kHz sinewave input with amplitude of 1 V. 8-fold oversampling is used.*

The second possibility is exploiting the output signal value $U_\mathrm{d}[n]$ for the computation of the filter coefficients. This algorithm is analogous to implicit methods for solving the ODEs. The output signal equation is in the following form:

$$\begin{aligned} U_\mathrm{d}[n] = {} & \frac{a_0(U_\mathrm{d}[n])}{b_0(U_\mathrm{d}[n])}\,U_\mathrm{i}[n] + \frac{a_1(U_\mathrm{d}[n])}{b_0(U_\mathrm{d}[n])}\,U_\mathrm{i}[n-1] - \\ & - \frac{b_1(U_\mathrm{d}[n])}{b_0(U_\mathrm{d}[n])}\,U_\mathrm{d}[n-1]. \end{aligned} \tag{8}$$

Substitution of filter coefficients from (5) and converting equation (8) into implicit form leads to the nonlinear function

$$\begin{aligned} f(\mathbf{U}_\mathrm{d}[n], \mathbf{U}_\mathrm{i}[n]) = {} & \frac{U_\mathrm{i}[n] + U_\mathrm{i}[n-1]}{\frac{I_\mathrm{s}}{U_\mathrm{d}}(\mathrm{e}^{\frac{U_\mathrm{d}[n]}{U_\mathrm{t}}} - 1)} - \\ & - \left(\frac{1 - 2f_\mathrm{s}CR}{\frac{I_\mathrm{s}}{U_\mathrm{d}}(\mathrm{e}^{\frac{U_\mathrm{d}[n]}{U_\mathrm{t}}} - 1)} + R \right) U_\mathrm{d}[n-1] - \\ & - \left(\frac{1 + 2f_\mathrm{s}CR}{\frac{I_\mathrm{s}}{U_\mathrm{d}}(\mathrm{e}^{\frac{U_\mathrm{d}[n]}{U_\mathrm{t}}} - 1)} + R \right) U_\mathrm{d}[n] = 0, \end{aligned} \tag{9}$$

where vector $\mathbf{U}_\mathrm{d}[n] = [U_\mathrm{d}[n], U_\mathrm{d}[n-1]]$ and $\mathbf{U}_\mathrm{i}[n] = [U_\mathrm{i}[n], U_\mathrm{i}[n-1]]$. The equation (9) is solved by the Newton method

$$U_\mathrm{d}^{k+1}[n] = U_\mathrm{d}^{k}[n] - \frac{f([U_\mathrm{d}^{k}[n], U_\mathrm{d}[n-1]], \mathbf{U}_\mathrm{i}[n])}{f'([U_\mathrm{d}^{k}[n], U_\mathrm{d}[n-1]], \mathbf{U}_\mathrm{i}[n])}, \tag{10}$$

where k is the iteration index and the value $U_\mathrm{d}^0[n]$ is the output signal estimation. The derivation f' is approximated by the finite difference formula. The output signal for the 10 kHz sinewave signal with an amplitude of 10 V and the time behavior of the filter coefficients are displayed in Figure 5.

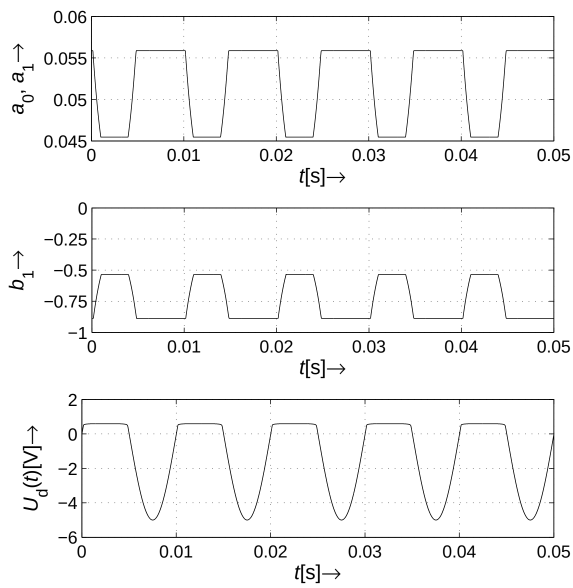


Figure 5: *Time behavior of coefficients and output signal when sinewave input signal is used.*

2.3. Reduction of iterations

The efficiency of the implicit type of the algorithm depends on the output signal estimation $U_{\mathrm{d}}^{0}[n]$. The efficient estimator is filter with coefficients computed in the last iteration

$$\begin{aligned} U_{\mathrm{d}}^{0}[n] = & \frac{a_0(U_{\mathrm{d}}[n-1])}{b_0(U_{\mathrm{d}}[n-1])} U_{\mathrm{i}}[n] + \\ & + \frac{a_1(U_{\mathrm{d}}[n-1])}{b_0(U_{\mathrm{d}}[n-1])} U_{\mathrm{i}}[n-1] - \\ & - \frac{b_1(U_{\mathrm{d}}[n-1])}{b_0(U_{\mathrm{d}}[n-1])} U_{\mathrm{d}}[n-1]. \end{aligned} \tag{11}$$

The output signal for the 10 kHz sinewave signal with an amplitude of 10 V and the number of iterations required for each output signal sample are shown in Figure 6. The estimated output signal value (solid line in Figures 7a and 7b) is the same as the computed value (crosses in Figures 7a and 7b) in the linear part of the transfer function, so the number of iterations here is one. A wrong estimation occurs in the nonlinear part of the transfer function (see Figure 7a) and the number of iterations rapidly grows here (dashed line in Figure 6). This could be solved by replacing the estimated value with

$$U_{\mathrm{dmax}}^{0}[n] = f(U_{\mathrm{imax}}), \tag{12}$$

where

$$f(U_{\mathrm{i}}) = RI_{\mathrm{s}}(\mathrm{e}^{\frac{U_{\mathrm{d}}}{U_{\mathrm{t}}}} - 1) + U_{\mathrm{d}} - U_{\mathrm{i}} \tag{13}$$

is the circuit equation without capacitor – the memory effect of the capacitor does not have any influence at the maximal saturation level. The result of this improvement is shown in Figure 7b and the number of iterations has been reduced (solid line in figure 6). The 8-fold oversampling has been used to reduce aliasing that can cause problems with stability at higher frequencies and also unwanted audio distortion.

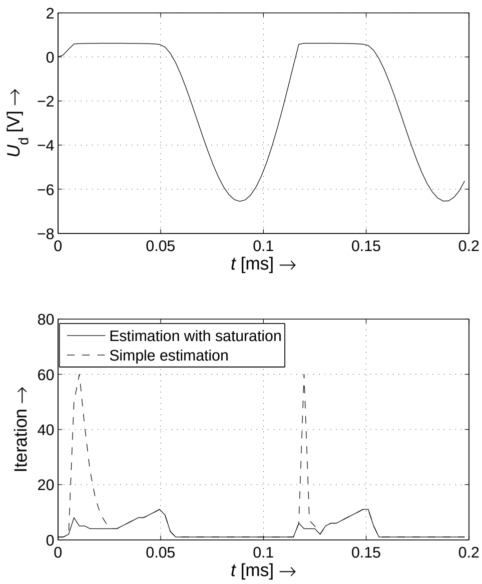
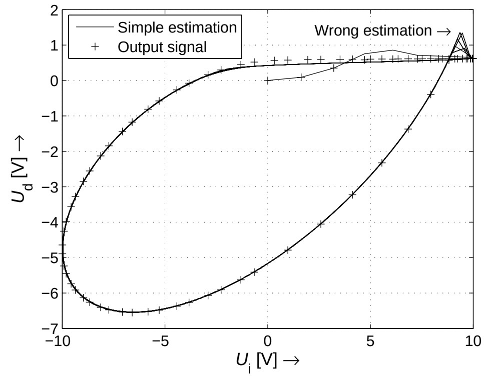
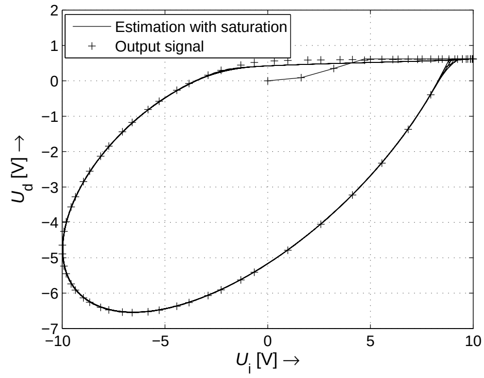


Figure 6: *Output signal (top) and required number of iterations per signal sample (bottom).*



(a)



(b)

Figure 7: *Dynamic transfer function of diode limiter and output estimation when simple filter (a) and filter with saturation (b) are used as the estimator. Solid line is the output signal estimation and crosses are the output signal values.*

2.4. Effect Implementation

The circuit from Figure 1 is a theoretical model. A circuit with two diodes connected in anti-parallel (see Figure 8) is used in real guitar distortion effect pedals. The nonlinear resistance r of both the diodes is

$$r = \frac{U_{\mathrm{d}}}{2I_{\mathrm{s}}\sinh(\frac{U_{\mathrm{d}}}{U_{\mathrm{t}}})} \tag{14}$$

and can be directly used in equation (5) to get the filter coefficients. The nonlinear output equation is now

$$\begin{aligned} f(\mathbf{U}_{\mathrm{d}}[n], \mathbf{U}_{\mathrm{i}}[n]) = & \frac{U_{\mathrm{i}}[n] + U_{\mathrm{i}}[n-1]}{\frac{2I_{\mathrm{s}}}{U_{\mathrm{d}}}\sinh(\frac{U_{\mathrm{d}}[n]}{U_{\mathrm{t}}})} - \\ & - \left(\frac{1 - 2f_{\mathrm{s}}CR}{\frac{2I_{\mathrm{s}}}{U_{\mathrm{d}}}\sinh(\frac{U_{\mathrm{d}}[n]}{U_{\mathrm{t}}})} + R \right) U_{\mathrm{d}}[n-1] - \\ & - \left(\frac{1 + 2f_{\mathrm{s}}CR}{\frac{2I_{\mathrm{s}}}{U_{\mathrm{d}}}\sinh(\frac{U_{\mathrm{d}}[n]}{U_{\mathrm{t}}})} + R \right) U_{\mathrm{d}}[n] = 0. \end{aligned} \tag{15}$$

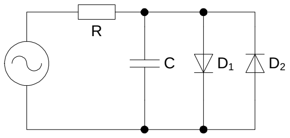


Figure 8: *Real diode limiter model.*

The whole algorithm is implemented according to the block diagram in Figure 9. The Nonlinear Solver block implements the Newton method for solving equation (15). The output signal value can be directly obtained from this block. It is possible to get the diode dynamic resistance according to (14) and then to compute filter coefficients that are used in the estimator. To prevent a higher number of iterations, the saturation block is connected after the estimator. The estimated value is then used in the Newton method in the next sample period.

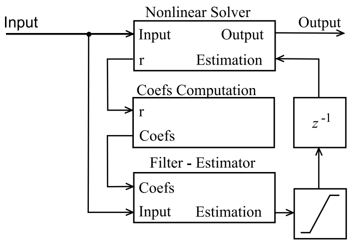


Figure 9: *Block diagram of the algorithm.*

This algorithm was implemented in Matlab and then tested on a real guitar signal and compared with Backward Euler method. An E chord guitar riff was used as the testing signal. The recorded signal was amplified to a maximum level of 10 Volts. In addition to the the output signal and difference between LTV filter and Euler mothod (see Figure 13), the average and maximum number of iterations in each sample period was examined, which is related to the computation demands of the algorithm. To avoid a dead-lock, the maximum of iterations was limited. The average number of iterations depends on the chosen numerical error (see Figure 10) – the value increases together with the accuracy of the algorithm. The number of iterations also depends on the level of the input signal, as is shown in Figure 11. The LTV filter method has the lower average number of iterations than Euler method, but the maximum number of iterations is higher (see Figure 12). Some problems with solution stability appeared during the testing of the algorithm when oversampling was not used. Therefore 4-fold oversampling has been used to ensure stability.

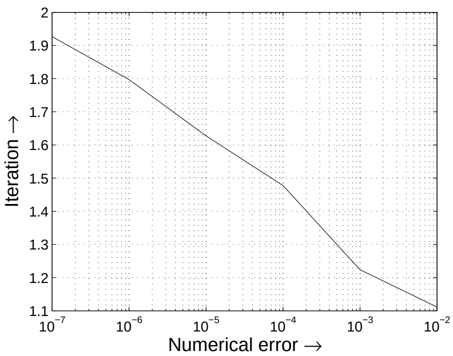


Figure 10: *Average number of iterations versus numerical error.*

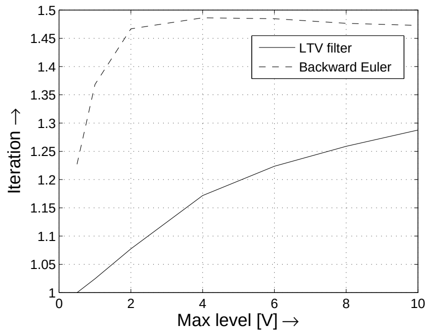


Figure 11: *Average number of iterations versus maximum input signal level. Numerical error is* 10^{-3}.

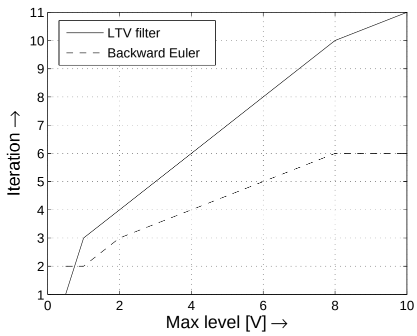


Figure 12: *Maximum number of iterations versus maximum input signal level. Numerical error is* 10^{-3}.

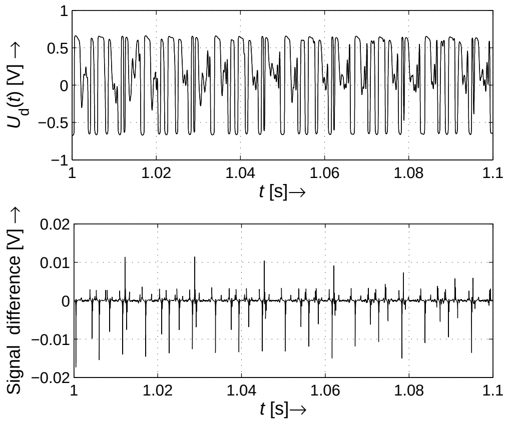


Figure 13: *Output signal for the part of guitar riff (top) and difference between LTV filter and Backward Euler method (bottom).*

3. CONCLUSIONS

The utilization of the linear time-variant digital filters for the simulation of nonlinear dynamic systems was discussed in this paper. Two types of algorithm were designed. The first one is a simple in-time iteration that requires very high sampling frequencies to get appropriate results. The second one is based on the Newton method and it can work at relatively low sampling frequencies. The efficiency of this algorithm depends on the estimation of the signal value in the next sample period. The digital filter seems to be an efficient estimator, especially in the linear parts of the transfer function. Saturation of the estimation was added to reduce the number of iterations in the nonlinear part of transfer function.

The designed algorithm was used to simulate the nonlinear circuit of the diode limiter and then tested on a real guitar signal. Some stability problems were observed when oversampling was not used. However, the algorithm gives good results when oversampling is applied and it is possible to use it in the guitar distortion effect.

The results of the algorithm were compared to the Backward Euler method. The both methods give almost the same output signal. Compared to the Euler method, the LTV filter method has the lower average number of iterations but the maximal number of iterations is higher.

4. ACKNOWLEDGMENTS

This paper was suported by the Czech Science Foundation (GACR) under grant project No. 102/07/P505 and 102/07/1303.

5. REFERENCES

[1] J. Macak, "Design of algorithms of digital audio processing for simulation of guitar combo based on circuit analysis of analogue prototypes (in czech)," M.S. thesis, Brno University of Technology, Brno, 2008.

[2] T. D. Yeh, J. S. Abel, and J. O. Smith, "Simplified, physically-informed models of distortion and overdrive guitar effects pedal," in *Proc. Digital Audio Effects (DAFx-07)*, Bordeaux, France, Sep. 10-15, 2007, pp. 189–196.

[3] T. D. Yeh, J. S. Abel, and J. O. Smith, "Simulation of the diode limiter in guitar distortion circuits by numerical solution of ordinary differential equations," in *Proc. Digital Audio Effects (DAFx-07)*, Bordeaux, France, Sep. 10-15, 2007, pp. 197–204.

[4] S. Barbati and T. Serafiny, "A Perceptual Approach on Clipping and Saturation," Available at http://www.simulanalog.org/clip.pdf, 2002.

[5] D. G. Manolakis and J. G. Proakis, *Digital Signal Processing: Principles, Algorithms and Applications*, Prentice Hall, Englewood Cliffs, NJ, USA, 3rd edition, 1991.

[6] A. Huovilainen, "Nonlinear digital implementation of the moog ladder filter," in *Proc. Digital Audio Effects (DAFx-04)*, Naples, Italy, Sep. 5-8, 2004, pp. 61–64.

[7] U. Zölzer, *DAFX - Digital Audio Effects*, J. Wiley & Sons, Ltd, 1st edition, 2002.